

CPT theorem in a $(5 + 1)$ Galilean space-time ¹

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Abstract

We extend the 5-dimensional Galilean space-time to a $(5+1)$ Galilean space-time in order to define a parity transformation in a covariant manner. This allows us to discuss the discrete symmetries in the Galilean space-time, which is embedded in the $(5 + 1)$ Minkowski space-time. We discuss the Dirac-type field, for which we give the 8×8 gamma matrices explicitly. We demonstrate that the CPT theorem holds in the $(5 + 1)$ Galilean space-time.

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1 Introduction

Galilean covariance is feasible in an enlarged $(4 + 1)$ -dimensional manifold, which is embedded in the $(4 + 1)$ Minkowski space-time [1, 2]. A 4-dimensional realization of the Clifford algebra in the $(4 + 1)$ Minkowski space-time requires γ^5 as a fourth “spatial” element of γ_μ s. In order to define a γ^5 -like matrix, it is necessary to extend the theories to a $(5 + 1)$ -dimensional manifold [3]. In this paper, we shall show that it is possible to define, in a covariant manner, the parity matrix, as well as the charge-conjugation and time-reversal matrices in a $(5 + 1)$ Galilean space-time. Hence we can discuss a CPT theorem based on the transformation properties of the Dirac-type field and state.

In the context of axiomatic field theory, the CPT theorem was established by Wightman [4] and his associates [5]. In relativistic quantum field theory, Schwinger proved the spin-statistics connection by symmetrizing (anti-symmetrizing) the kinematical term of the Lagrangian and, hence, the commutation (anticommutation) relations [6, 7]. Recently, Puccini and Vucetich axiomatized Schwinger’s Lagrangian formulation and proved the CPT theorem by assuming a form of dynamical Lagrangian [8]. Weinberg adopted the causality requirement, originally proposed by Pauli [9], to prove the connection between spin and statistics as well as the CPT theorem [4].

In Ref. [3], we have developed an 8-dimensional realization of the Clifford algebra in the $(4 + 1)$ Galilean space-time, in order to define the discrete symmetries, in particular, the parity transformation. This was accomplished by using the dimensional reduction from the $(5 + 1)$ Minkowski space-time to the $(4 + 1)$ Minkowski space-time, which, when expressed in terms of light-cone coordinates, corresponds to the $(4 + 1)$ Galilean space-time.

In order to avoid using projective representations of the Galilei group, which arise from the existence of a central charge, the $(4 + 1)$ Galilean space-time was introduced as the realization of a central extension of the group. The usual $(3 + 1)$ space-time is embedded into the $(4 + 1)$ Galilean space-time. In odd-dimensional Minkowski space-times, in which the number of spatial coordinates is even, the reflection of spatial manifold has a determinant equal to one, so that it is continuously connected to the identity and can be obtained as a rotation.

Parity refers to a reversal of orientation of the spatial coordinates. Thus we define the parity transformation by the mapping:

$$P' : \quad x^\mu \rightarrow x'^\mu = (-\mathbf{x}, x^4, x^5).$$

However, the existence of this discrete transformation entails the loss of manifest covariance, because the space reflection in the $(4 + 1)$ Minkowski space-time corresponds to

$$P : \quad x^\mu \rightarrow x'^\mu = (-\mathbf{x}, x^5, x^4).$$

A way to preserve both manifest covariance and the discrete parity operation is to extend the $(4 + 1)$ Galilean space-time to a $(5 + 1)$ Galilean space-time. The latter space-time corresponds to a $(5 + 1)$ Minkowski space-time defined with light-cone coordinates. Motivated by this fact we develop, in this paper, a 6-dimensional Galilean theory in a covariant manner and prove the CPT theorem.

For the $(5+1)$ Galilean space-time, we use light-cone coordinates x^μ ($\mu = 1, \dots, 6$), with the metric tensor

$$\eta_{\mu\nu} = \begin{pmatrix} 1_{4 \times 4} & 0_{4 \times 2} \\ 0_{2 \times 4} & \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} \end{pmatrix}.$$

Then the coordinate system y^μ ($\mu = 1, \dots, 5, 0$), defined by

$$\mathbf{y} = \mathbf{x}, \quad y^4 = x^4, \quad y^5 = \frac{1}{\sqrt{2}}(x^5 - x^6), \quad y^0 = \frac{1}{\sqrt{2}}(x^5 + x^6), \quad (1)$$

admits the diagonal metric

$$g_{\mu\nu} = \text{diag}(1, 1, 1, 1, 1, -1).$$

This correspondence between the $(5+1)$ Galilean space-time and the $(5+1)$ Minkowski space-time allows us to describe non-relativistic theories in a Lorentz-like covariant manner [1, 2].

Let Γ^μ and γ^μ denote the 8×8 gamma matrices in the $(5 + 1)$ Galilean (light-cone coordinates xs) and Minkowski space-times (coordinates ys), respectively. The gamma matrices transform as contravariant vectors in each space-time. Therefore, we have

$$\Gamma^k = \gamma^k = \begin{pmatrix} 0_{4 \times 4} & 0 & \sigma_k \\ & -\sigma_k & 0 \\ 0 & \sigma_k & \\ -\sigma_k & 0 & 0_{4 \times 4} \end{pmatrix}, \quad k = 1, 2, 3,$$

$$\Gamma^4 = \gamma^4 = \text{i} \begin{pmatrix} 0_{4 \times 4} & 0 & I \\ & I & 0 \\ 0 & -I & \\ -I & 0 & 0_{4 \times 4} \end{pmatrix},$$

$$\Gamma^5 = \frac{1}{\sqrt{2}}(\gamma^5 + \gamma^0) = -\sqrt{2} \text{i} \begin{pmatrix} 0_{4 \times 4} & 0 & 0 \\ & 0 & I \\ I & 0 & \\ 0 & 0 & 0_{4 \times 4} \end{pmatrix},$$

$$\Gamma^6 = \frac{1}{\sqrt{2}}(-\gamma^5 + \gamma^0) = -\sqrt{2} \text{i} \begin{pmatrix} 0_{4 \times 4} & I & 0 \\ & 0 & 0 \\ 0 & 0 & \\ 0 & I & 0_{4 \times 4} \end{pmatrix},$$

$$\zeta = \frac{1}{\sqrt{2}}i(\Gamma^5 + \Gamma^6) = i\gamma^0 = \begin{pmatrix} & I & 0 \\ 0_{4 \times 4} & 0 & I \\ I & 0 & \\ 0 & I & 0_{4 \times 4} \end{pmatrix}, \quad (2)$$

$$\Gamma^7 = \gamma^7 = \gamma^1 \gamma^2 \gamma^3 \gamma^4 \gamma^5 \gamma^0 = \begin{pmatrix} I & 0 & & \\ 0 & I & & 0_{4 \times 4} \\ & & -I & 0 \\ 0_{4 \times 4} & & 0 & -I \end{pmatrix},$$

where

$$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Note that we have inverted γ^4 and γ^5 with respect to Ref. [3].

The canonical conjugate variable of the extended coordinates in the (5+1) Galilean space-time provides a transparent interpretation of the additional parameter s . Indeed, the 6-momentum,

$$\begin{aligned} p_\mu = -i\partial_\mu &= (-i\nabla, -i\partial_4, -i\partial_t, -i\partial_s), \\ &= (\mathbf{p}, p_4, -E, -m), \end{aligned}$$

such that $p^5 = -p_6 = m$ and $p^6 = -p_5 = -E$, shows that the coordinate s is conjugate to the mass m in the same way that \mathbf{x} is conjugate to the momentum \mathbf{p} .

2 Discrete symmetries

The parity and time-reversal transformations in the (5+1) Minkowski space-time are defined respectively by

$$P : y^\mu \rightarrow y'^\mu = (-\mathbf{y}, -y^4, -y^5, y^0) = -g_{\mu\nu}y^\nu,$$

$$T : y^\mu \rightarrow y'^\mu = (\mathbf{y}, y^4, y^5, -y^0) = g_{\mu\nu}y^\nu,$$

which correspond to

$$P : x^\mu \rightarrow x'^\mu = (-\mathbf{x}, -x^4, x^6, x^5) = -\eta_{\mu\nu}x^\nu, \quad (3)$$

$$T : x^\mu \rightarrow x'^\mu = (\mathbf{x}, x^4, -x^6, -x^5) = \eta_{\mu\nu}x^\nu, \quad (4)$$

in the (5+1) Galilean space-time. The transformation matrices for parity and time reversal, denoted by R_A ($A = P, T$), are defined by imposing that the Dirac (and Dirac-like) equation be invariant under the transformations $A = P, T$. The Dirac

equation and the Dirac-like equation are equivalent to each other under the conditions (1), that is,

$$-(i\gamma \cdot \hat{\partial} + \kappa_m)\hat{\psi}(y) = -(i\Gamma \cdot \partial + \kappa_m)\psi(x) = 0,$$

where

$$\hat{p}_\mu = -i\hat{\partial}_\mu = -i\frac{\partial}{\partial y^\mu},$$

and

$$\hat{p}_\mu \hat{p}^\mu = p_\mu p^\mu = -\kappa_m^2,$$

with

$$\kappa_m = \sqrt{2} m.$$

This leads to the fact that if transformation properties under $A = P, T$ are obtained in one 6-dimensional space-time, then the same transformation properties hold in the other 6-dimensional space-time, in which the metric tensors are interchanged: $g_{\mu\nu} \leftrightarrow \eta_{\mu\nu}$. Henceforth, we discuss the transformation properties in the $(5 + 1)$ Galilean space-time.

The transformation matrices R_A (where $A = P, T$) are obtained by imposing

$$-R_A^{-1} (\Gamma \cdot \partial' + \kappa_m) R_A = -(\Gamma \cdot \partial + \kappa_m), \quad A = P, T,$$

where

$$\begin{aligned} R_P^{-1} &= \zeta^{-1} R_P^\dagger \zeta, \\ R_T^{-1} &= -\zeta^{-1} R_T^\dagger \zeta, \end{aligned} \tag{5}$$

and ζ , defined as in Eq. (2), satisfies $\zeta = \zeta^{-1}$. The notation ∂' is defined by

$$\partial'_\mu = \frac{\partial}{\partial x'^\mu},$$

where x'^μ is given by Eq. (3) or (4), depending on whether $A = P$ or T . The negative sign in the second expression of Eq. (5) arises from the fact that Dirac-type (Dirac) field should obey anti-commutation relations. It is important to mention that for a scalar-type (scalar) field and a vector-type (vector) field, this sign will be positive.

Explicitly, we find that

$$\begin{aligned} R_P &= \gamma^0 = \frac{1}{\sqrt{2}}(\Gamma^5 + \Gamma^6), \\ R_T &= \gamma^1 \gamma^2 \gamma^3 \gamma^4 \gamma^5 = \gamma^0 \gamma^7 = \frac{1}{\sqrt{2}}(\Gamma^5 + \Gamma^6) \Gamma^7. \end{aligned}$$

The charge-conjugation matrix C is defined by

$$(\Gamma^\mu)^T = -C^{-1} \Gamma^\mu C = \hat{C}^{-1} (\Gamma^\mu)^\dagger \hat{C}, \tag{6}$$

with

$$\hat{C}^\dagger = \hat{C}^{-1} = -\hat{C}^*, \quad \hat{C}^T = -\hat{C},$$

where

$$\begin{aligned}\hat{C} &= \gamma^0 C = \frac{1}{\sqrt{2}}(\Gamma^5 + \Gamma^6)C, \\ (\Gamma^\mu)^\dagger &= \frac{1}{2}(\Gamma^5 + \Gamma^6)\Gamma^\mu(\Gamma^5 + \Gamma^6).\end{aligned}\tag{7}$$

The charge-conjugation matrix \hat{C} satisfies

$$[-\zeta (\Gamma \cdot \partial + \kappa_m)]^T = \{-\zeta [\Gamma \cdot (-\partial) + \kappa_m]\}^* = \hat{C}^{-1} \zeta [\Gamma \cdot (-\partial) + \kappa_m] \hat{C},$$

which reduces to

$$-C[\Gamma \cdot (-\partial) + \kappa_m]^T C^{-1} = -(\Gamma \cdot \partial + \kappa_m).$$

Galilean theory is manifestly covariant under Galilean transformations. Therefore equations remain valid in all frames of reference. This statement does not make sense, however, unless transformations are adequately assigned to the fields. Thus, hereafter we define such field transformations.

Now let us define the C , T and P transformation properties of Dirac-type fields. For charge conjugation C , we have:

$$\begin{aligned}\psi'(x') &= \xi_C \psi_C(x) = \xi_C \hat{C} \psi^*(x), \\ \bar{\psi}'(x') &= \psi'^\dagger(x') \zeta = \xi_C^* \bar{\psi}_C(x) = \xi_C^* \psi^T(x) \hat{C}^\dagger \zeta,\end{aligned}\tag{8}$$

with

$$\hat{C} : \Psi \rightarrow \Psi' = \Psi,$$

where the star $*$ denotes the complex conjugation of a c -number (including gamma matrices) and the Hermitian adjoint of an operator. For time-reversal T , we define

$$\begin{aligned}\psi'(x') &= \xi_T R_T \tilde{\psi}_C(x), \\ \bar{\psi}'(x') &= \xi_T^* \tilde{\psi}_C^T(x) \zeta^{-1} R_T^\dagger \zeta,\end{aligned}\tag{9}$$

with

$$T : \Psi \rightarrow \Psi' = \Psi^*,$$

where the tilde \sim denotes the transposition of an operator in $\psi_C(x)$. For a parity transformation P , we have

$$\begin{aligned}\psi'(x') &= \xi_P R_P \psi(x), \\ \bar{\psi}'(x') &= \xi_P^* \bar{\psi}(x) \zeta^{-1} R_P^\dagger \zeta,\end{aligned}\tag{10}$$

with

$$P : \Psi \rightarrow \Psi' = \Psi.$$

Here Ψ stands for the state vector and the complex c -number ξ_A (where $A = C, P, T$) can be normalized to unity without loss of generality:

$$|\xi_A|^2 = \xi_A \xi_A^* = 1.$$

Now that we have assigned the transformation properties of the discrete symmetries to the fields, the transformations of the bilinear forms under \hat{C} , P and T will be obtained below.

2.1 Charge conjugation

Recalling the definitions of C and \hat{C} in Eqs. (6) to (7), we find

$$\bar{\psi}'(x')\Gamma^A\psi'(x') = -\epsilon^A\bar{\psi}(x)\Gamma^A\psi(x),$$

with

$$\epsilon^A = \begin{cases} +1, & \text{for } \Gamma^A = 1, \Gamma^7\Sigma^{\mu\nu}, \Sigma^{\lambda\mu\nu}, \\ -1, & \text{for } \Gamma^A = \Gamma^7, \Gamma^\mu, i\Gamma^7\Gamma^\mu, \Sigma^{\mu\nu}, \end{cases}$$

where

$$\begin{aligned} \Sigma^{\mu\nu} &= \frac{1}{2i}(\Gamma^\mu\Gamma^\nu - \Gamma^\nu\Gamma^\mu), \\ \Sigma^{\lambda\mu\nu} &= \frac{1}{3}(\Gamma^\lambda\Sigma^{\mu\nu} + \Gamma^\mu\Sigma^{\nu\lambda} + \Gamma^\nu\Sigma^{\lambda\mu}). \end{aligned}$$

Here we have utilized the relationship:

$$\zeta^{-1}(\hat{C}^\dagger\zeta\Gamma^A\hat{C})^T = -(C^{-1}\Gamma^AC)^T = -\epsilon^A\Gamma^A.$$

The commutation relation becomes

$$\{\psi_{C\alpha}(x_1), \bar{\psi}_C^\beta(x_2)\} = i d_\alpha{}^\beta(\partial_1)\Delta(x_1 - x_2), \quad (11)$$

where

$$d(\partial) = -(\Gamma \cdot \partial - \kappa_m).$$

The commutation relation in Eq. (11) is invariant under the charge-conjugation transformation of Eq. (8). Therefore, there exists a unitary transformation which is called the ‘charge-parity operator’, such that

$$\begin{aligned} G_C^{-1}\psi(x)G_C &= \xi_C\hat{C}\psi^*(x), \\ G_C^{-1}\bar{\psi}(x)G_C &= \xi_C^*\psi^T(x)\hat{C}^\dagger\zeta, \end{aligned}$$

with

$$G_C|0\rangle = |0\rangle.$$

2.2 Time reversal

The bilinear forms transform under the time-reversal transformation, Eq. (9), as

$$\begin{aligned} \bar{\psi}'(x')\Gamma^A\psi'(x') &= -\tilde{\bar{\psi}}(x)\zeta^{-1}(\hat{C}^\dagger\zeta R_T^{-1}\Gamma^AR_T\hat{C})^T\tilde{\psi}(x), \\ &= \epsilon^T\epsilon^A\tilde{\bar{\psi}}(x)\Gamma^A\tilde{\psi}(x), \end{aligned}$$

with

$$\epsilon^T = \begin{cases} 1, & \text{for } \Gamma^A = 1, \\ -1, & \text{for } \Gamma^A = \Gamma^7, \\ \eta_{\mu\rho}, & \text{for } \Gamma^A = \Gamma^\mu, \\ -\eta_{\mu\rho}, & \text{for } \Gamma^A = i\Gamma^7\Gamma^\mu, \\ \eta_{\mu\rho}\eta_{\nu\sigma}, & \text{for } \Gamma^A = \Sigma^{\mu\nu}, \\ -\eta_{\mu\rho}\eta_{\nu\sigma}, & \text{for } \Gamma^A = \Gamma^7\Sigma^{\mu\nu}, \\ \eta_{\lambda\rho}\eta_{\mu\sigma}\eta_{\nu\tau}, & \text{for } \Gamma^A = \Sigma^{\lambda\mu\nu}, \end{cases} \quad (12)$$

which reads, for example,

$$\bar{\psi}'(x')\Gamma^\mu\psi'(x') = \eta_{\mu\rho}(-1)\tilde{\bar{\psi}}(x)\Gamma^\rho\tilde{\psi}(x).$$

It should be remarked that a physical quantity A , expressed in functional form, is invariant under the time-reversal transformation in the sense:

$$A[\psi'(x'), \partial'_\mu\psi'(x'), \dots] = \tilde{A}[\psi(x), \partial_\mu\psi(x), \dots].$$

The commutation relation is obtained from Eqs. (9) and (11):

$$\{\psi'_\alpha(x'_1), \bar{\psi}'^\beta(x'_2)\} = i d_\alpha{}^\beta(\partial'_1) \Delta(x'_1 - x'_2).$$

The invariance of the commutation relations implies that a unitary transformation exists:

$$\begin{aligned} G_T^{-1}\psi(x)G_T &= \xi_T R_T \tilde{\psi}_C(x'), \\ G_T^{-1}\bar{\psi}(x)G_T &= \xi_T^* \tilde{\bar{\psi}}_C(x') (\zeta^{-1} R_T^\dagger \zeta), \end{aligned}$$

with

$$G_T|0\rangle = |0\rangle,$$

and x' given by Eq. (4).

If we define

$$\Psi^T = G_T \Psi^* = G_T \Psi',$$

then

$$\langle \Psi'_2, G_T^{-1}\psi(x)G_T \Psi'_1 \rangle = \langle \Psi_2^T, \psi(x)\Psi_1^T \rangle = \langle \Psi_2^*, \tilde{\psi}(x')\Psi_1^* \rangle = \langle \Psi_1, \psi(x')\Psi_2 \rangle.$$

2.3 Space reflection

The bilinear forms transform under the space reflection transformation, Eq. (10), as

$$\begin{aligned} \bar{\psi}'(x')\Gamma^A\psi'(x') &= \bar{\psi}(x)R_P^{-1}\Gamma^A R_P\psi(x), \\ &= \epsilon^P \bar{\psi}(x)\Gamma^A\psi(x), \end{aligned}$$

where ϵ^P is obtained by the replacement

$$\eta_{\mu\nu} \rightarrow -\eta_{\mu\nu},$$

in ϵ^T of Eq. (12). This implies the important fact that vectors and rank-3 tensors change signs, whereas the signs of other quantities remain unchanged under successive discrete transformations \hat{C} , T and P . Thus, the CPT theorem holds in the $(5+1)$ Galilean space-time.

The commutation relation is expressed in the form

$$\{\psi'_\alpha(x'_1), \bar{\psi}'^\beta(x'_2)\} = i d_\alpha{}^\beta(\partial'_1)\Delta(x'_1 - x'_2),$$

which is invariant under the parity transformation, Eq. (10). This means that a unitary transformation G_P (the parity operator) exists, such that

$$\begin{aligned} G_P^{-1}\psi(x)G_P &= \xi_P R_P \psi(x'), \\ G_P^{-1}\bar{\psi}(x)G_P &= \xi_P^* \bar{\psi}(x') R_P^{-1}, \end{aligned}$$

with

$$G_P|0\rangle = |0\rangle,$$

and x' given in Eq. (3). In deriving the relations developed in this section, the vacuum subtraction should be understood.

As stated earlier, we can develop the same procedure in the $(5+1)$ Minkowski space-time and the same results are obtained by replacing $\eta_{\mu\nu}$ with $g_{\mu\nu}$. The Dirac gamma matrices are given by using the same charge-conjugation matrix as in Eq. (6):

$$\gamma^A = \epsilon^A (C^{-1}\gamma^A C)^T,$$

with

$$\epsilon^A = \begin{cases} +1, & \text{for } \gamma^A = 1, \gamma^7 \sigma^{\mu\nu}, \sigma^{\lambda\mu\nu}, \\ -1, & \text{for } \gamma^A = \gamma^7, \gamma^\mu, i\gamma^7 \gamma^\mu, \sigma^{\mu\nu}, \end{cases}$$

where

$$\begin{aligned} \sigma^{\mu\nu} &= \frac{1}{2i}(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu), \\ \sigma^{\lambda\mu\nu} &= \frac{1}{3}(\gamma^\lambda \sigma^{\mu\nu} + \gamma^\mu \sigma^{\nu\lambda} + \gamma^\nu \sigma^{\lambda\mu}). \end{aligned}$$

3 Concluding remarks

In relativistic quantum field theory, the CPT theorem was developed in connection with spin and statistics [11, 12]. Lüders gave a proof of the CPT theorem by using the transformation properties of each kind of field and hence a connection between spin and statistics was postulated [12]. In fact, he made the proof for the fields with spin 0, 1/2 and 1, and concluded that the proof can be extended to fields with arbitrary spin, which are constructed in terms of spin 1/2 and 1 fields.

The conclusion that the CPT theorem holds in the $(5+1)$ Galilean space-time is easily extended to the scalar- and vector-type fields, since the proof given here is based on the transformation properties of the field. Thus the CPT theorem in $(5+1)$ Galilean space-time holds, in the sense of Lüders. Furthermore it may be stated, following Schwinger [6], that “connection between spin and statistics of particles is implicit in the requirement of invariance under coordinate transformations”.

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